RESULTS OF MATHEMATICAL MODELING OF A HEAT-CONDUCTION PROCESS

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Using a special regularizing algorithm, the induction quenching of steel samples is mathematically modeled, allowing optimal conditions for the process to be chosen.

1. The modern level of development of computational techniques is such as to allow apparatus state sensors to be replaced by program sensors in many cases of technological processes, which extends the possibilites of solving control problems for industrial equipment. To this end, mathematical models which reflect the basic features of the physics of the process are being developed; the purpose of the program is to output certain characteristics of the result. However, the formulation of the control problem assumes that the function really controlling the process is unknown and must be determined from the target characteristics. In this case some inverse problem is an element of the mathematical model, and the creation of a program sensor requires the use of Tikhonov [1] regularizing algorithms.

These questions will be considered in the present work for the example of modeling the inductional quenching of long cylindrical samples, as a result of which certain minute effects in the behavior of the process output characteristics may be observed.

Note that intrinsically inverse problems in heat-conduction theory have been considered in a sufficiently broad circle of works, for example, [2-5], where their correct formulation and regularizing algorithms were developed. However, in those cases where an inverse problem is an element of a modeling problem, the question of the economy of the algorithm arises, and the evolutionary character of the heat-conduction processes allows, as will be shown below, special algorithms of this type to be developed [6].

2. In the induction quenching of steel samples by high-frequency currents, there are three stages of the process: a) fast heating to a temperature of $\sim 1000°C$; b) more or less prolonged isothermal holding; c) fast cooling by a fluid flow immersing the surface. For the quenching of long cylindrical samples, the desired behavior of the surface temperature is the target characteristic for heating in a longitudinal magnetic field (Fig. 1). However, it cannot be specified by means of boundary conditions in formulating the problem, not only because the temperature, strictly speaking, is not realized, but mainly because volume heating is used, and the source density depends on the really controlling function $\chi(t)$, the magnitude of the magnetic field at the surface of the sample, which is proportional to the current intensity in the induction loop for a small gap, and so itself must be determined from the specified $\varphi(t)$. Thus, modeling the first two stages involves the simultaneous determination of the temperature field and the function $\chi(t)$.

In this case the temperature field is described by a system of Maxwell and heat-conduction equations; these are nonlinear, since the thermal and electromagnetic characteristics of the material are also functions of the temperature. Their behavior on heating is known [7]. Methods of solving the boundary problems for such systems on a computer without any simplifying assumptions with regard to the behavior of the coefficients are well developed [8, 9], and the general algorithm for solving this system on any time segment $[t^{(1)}, t^{(2)}]$ for arbitrary initial and physically correct boundary conditions

$$-k(u)\frac{\partial u}{\partial r}\bigg|_{r=R} = \Psi(u), \tag{1}$$

where $\Psi(u)$ a specified function is assumed to be known. For each $\chi(t)$, the temperature field is determined everywhere, including the sample surface: $u(R, t) = v[t, \chi]$.

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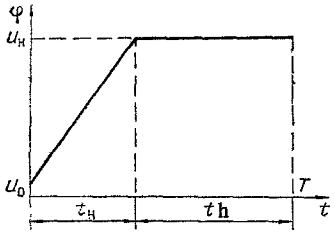

Fig. 1. Sample surface temperature for heating t_H and isothermal holding t_h.

The corresponding algorithm was realized in [9] within the framework of the two-dimensional model (r, t) and the experimentally verified (for fast processes) assumption of negligible heat transfer to the sample surface; $\Psi(u) \equiv 0$ on heating.

The following algorithm of successive conditional minimization of the norm of the deviation of v[t, χ] from the specified $\varphi(t)$ is then realized. The complete-heating time interval [0, T) is divided into parts: $[t_{s-1}, t_s]$, $s = 1, 2, \ldots, n$ $(t_0 = 0)$* and, correspondingly,† $\chi(t)$ is replaced by the piecewise constant function $\{\chi_s\}$. For sufficiently large n, this approximation is as accurate as could be wished, and $\{\chi_s\}$ may be found from a sequence of variational problems, each of which is written for only one variable

$$\min(\chi - \chi_{s-1})^2, \quad \chi \in X_{s,\delta} \equiv \{\chi : \|v_s[t, \chi] - \tilde{\varphi}_s\|_s^2 \leqslant \delta_s^2\}, \tag{2}$$

where $\|v[t, \chi] - \tilde{\varphi}\|_s^2 \equiv \int_{t_{s-1}}^{t_s} \{v_s[t, \chi] - \tilde{\varphi}(t)\}^2 dt$; δ is a measure of the deviation from the "desired" $\varphi(t)$ permitted in the given interval; $v_s[t, \chi]$ is determined by the direct problem described above for the segment $[t_{s-1}, t_s]$ with the field-continuity condition at the point t_{s-1} (initial). This algorithm is similar to the dynamic-programming scheme for control problems [10], accurate up to terms in the operator v[t, χ] represented in this case by a system of partial differential operators, the difference being that here only a direct approach to variational search (from 0 to T) is used, and stability is achieved as a result of the introduction of a sequence of local regulators. This determines its economy.

The proposed algorithm R_p^δ is substantiated as Tikhonov conditionally regularizing. It has been established, for example, that under conditions corresponding to the considered "direct" problem (in particular, in view of the maximum principle for the heat-conduction equations), and provided the specified $\varphi(t)$ corresponds to the unique "accurate" controlling function $\{\hat{\chi}_s\}$, then for any measure of the error $\delta : \|v[t, \chi] - \varphi(t)\|_{[0, T]} \leq \delta$, there exists a sequence $\{\delta_s\}$ $(\delta_s \leq \delta/n)$ such that the sequence in Eq. (2) has a solution $\{\tilde{\chi}_s\}$ and $\max|\tilde{\chi}_s - \hat{\chi}_s| \to 0$ as $\delta \to 0$.

In [6], it was shown on models that R_p^δ is efficient in the class of piecewise-smooth functions with a sufficiently sharp maximum, which are characteristic of the control problems considered, if the transition from rapid rise in temperature through the Curie point to maintenance of the temperature at a constant level is taken into account.

Obviously, the temperature field in each subsequent $[t_{s-1}, t_s]$ is calculated simultaneously with χ_s, and is fixed as soon as χ_s has been found with the required accuracy.

In part I of Fig. 2, results calculated for the controlling function $\chi(t)$, and simultaneously the temperature field, using R_p^δ, are shown in comparison with experimental data for one set of quenching conditions. The calculations assume $n = 177$, $\delta_s^2 = 10^{-2}$.

The series of these, and subsequent, calculations provides the basis for conclusions regarding the target specification of the considered technological process (Sec. 4).

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*This division does not coincide with the difference-grid step used in numerical solution of the direct problem [9].

†This replacement does not necessarily mean approximation of $\chi(t)$, since discrete control in the induction is possible.

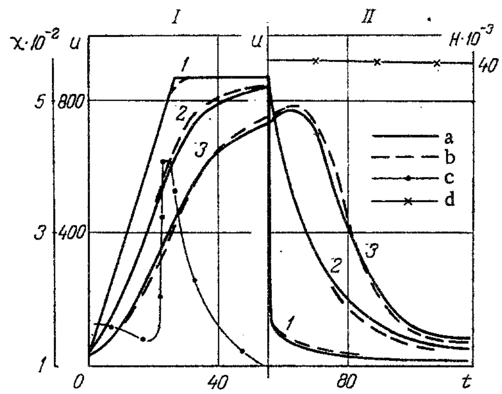

Fig. 2. Temperature field u (°C), magnetic field at surface χ (A/mm), and convective-heat-transfer coefficient H (kcal/m · h · deg) for heating and holding (I) and cooling (II): a) calculation; b) experiment; c) χ; d) H; 1) surface; 2) r = 16.5 mm; 3) center of sample, St. 40; R = 24 mm, t_H = 27 sec, t_h = 28 sec, u_H = 870°C, f = 2500 Hz.

3. To develop a model of the process at the fast-cooling stage, when the temperature dependence of the material's thermal characteristics obtained in slow processes does not correspond to the physical reality [11], and various hypotheses regarding the heat-transfer law at the surface are possible (film or bubble boiling, convective heat transfer [12]), a mathematical experiment was performed on a computer. For each of the two possible hypotheses regarding the behavior of k(u) and $c\gamma(u)$, the algorithm R_p^δ was used to solve the inverse problem for H*(t), which characterizes the heat-transfer law at the surface. Additional information was specified here, in the form of the real surface temperatures: $\tilde{\varphi}(t)$ obtained experimentally [13] with a certain error δ. The heat-transfer condition in Eq. (1) was written for $\Psi(u) \equiv H(u)(u-u_0)$, where u_0 is the temperature of the surrounding medium (~ 25°C), and since the surface temperature is known the following relation was established: H(u) = H*(t). Obviously, the sequence of problems in Eq. (2) may also be written in the case with the substitution $\chi_s \to H_s$, but the operator v[t, H] is then defined by a homogeneous nonlinear heat-conduction equation.

As a result of the experiment, it was found possible to choose a phenomenological model of the cooling process within the framework of which the thermal characteristics remain constant in layers heated up to and above the austenitic-transformation temperature, right up to the structural transformation to martensite [11]. Then convective heat transfer with the medium occurs at the surface, which is physically verified at large velocities of the cooling liquid bathing the sample. In part II of Fig. 2, the dependence H*(t) obtained for the same parameters of the process as above is shown, together with the temperature field and a comparison with experiment.

Note that the other hypothesis, that the thermal characteristics in all the layers are the same as in heating, although it gives some "smooth" dependence H*(t) differing from a constant, leads to sharp divergence of the temperature field from the results of physical experiment.

The computer mathematical experiment, based on the use of a Tikhonov regularizing algorithm for the solution of the corresponding inverse problem, allows the construction of the quenching model to be improved.

4. The program calculated within the framework of the model developed above may serve as the state-characteristic sensor of the object of a technological process, specifically, the temperature field and the controlling magnetic field of the inductor. With sufficient provision of the process with computational techniques, this sensor may serve as the object of the dynamic equation of heating under quenching.

On the other hand, treating the data on the temperature field in dynamic conditions allows definite efficiency characteristics of the process to be obtained. These characteristics will be taken to be the calcination depth (the thickness of the surface layer) Δ^i with a specified i-percent martensite content and the "effective hardness" of the sample [14] θ (the statistical moment for the torsional forces). Each will be regarded as a function of the holding time for a specified time of rapid heating and various values of the rate of surface cooling (different H_0): $\Delta^i = \Delta^i(t_h)$, $\theta = \theta(t_h)$. Since the phase-state diagrams of carbon steel are known [15], the dis-

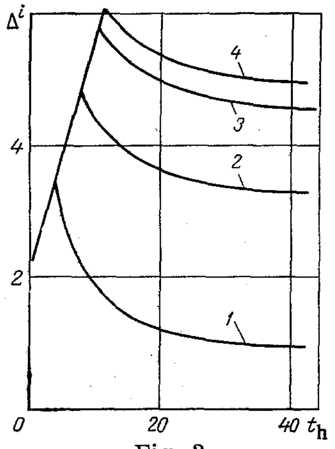

Fig. 3

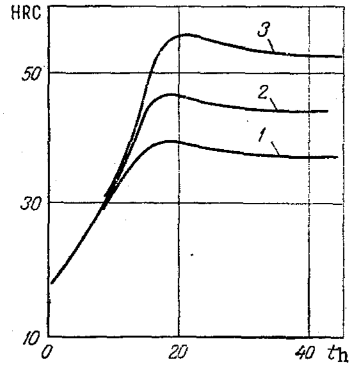

Fig. 4

Fig. 3. Thickness of semimartensite structure Δ^i, mm ($i = 50\%$); 1) $H_0 =$ 5000 kcal/m·h·deg; 2) 12,000; 3) 40,000; 100,000; St. 45; R = 20 mm; $t_H = 4$ sec; $u_H = 880°C$; f = 2400 Hz.

Fig. 4. Effective hardness of sample: 1) $H_0 = 5000$ kcal/m·h·deg; 2) 1200; 3) 100,000; St. 45; R = 20 mm; $t_H = 4$ sec; $u_H = 880°C$; f = 2400 Hz.

tribution of the percentage austenite content over the sample cross section at the end of heating may be found from the known temperature field at this time. In turn, knowing the cooling rate of each sample layer at any moment of time, it is possible, using the thermokinetic supercooled-austenite decay curves (C-shaped curves [16]), to determine the percentage martensite content in each layer of the sample as a result of cooling for each set of parameters characterizing the prevailing conditions, and thereby to find $\Delta(t_h)$. Since the corresponding treatment can be completely algorithmized and, hence, may be automated on a computer, this program serves as the sensor for the quenching efficiency index.

One of the nomograms obtained for $\Delta^i(t_h)$ when $i = 50\%$ for different H_0 and a certain set of other parameters of the sample and the quenching conditions is shown in Fig. 3. The effects observed in Fig. 3 are that: a) there exists a limiting calcination depth with rise in holding time, so that increase in the latter, which facilitates through heating of the sample, does not lead to increase in quenched-layer thickness: on surface cooling, the heat flux internal to the surface, which rises with increase in heating depth, reduces the cooling rate in the inner layer of the sample; b) there exists a maximum value of $\Delta^i(t_h)$ which is reached at relatively short holding times. This is because the above-mentioned physical process "competes" with the process of increase in heating depth, which rises with rise in t_h [6]. The nomograms obtained give a quantitative estimate of this effect, and serve as the object of the increase in efficiency of the technological process of quenching.

In Fig. 4, nomograms are shown for $\theta(t_h)$ for the same parameters of the process. These nomograms are obtained as a consequence of the preceding, since the static moment in uniquely established from the hardness distribution over the cross section, and the latter depends on the percentage content of different phases in each layer, determined in the preceding state of the analysis. The shift in the maximum to the right on the curves shown occurs because the internal heat fluxes which impede the growth in quenched-layer thickness also facilitates a smoother distribution of martensite structure over the layer (decrease in "curvature" of the hardness curves). Estimation of the position of the maximum or the value of t_h at which the required value of θ is obtained, $\theta_H < \theta_{max}$, as derived by programming methods, is also an aim of economizing the technological process.

LITERATURE CITED

1. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Incorrect Problems [in Russian], Nauka, Moscow (1974).
2. A. N. Tikhonov and V. B. Glasko, "Methods of determining body surface temperatures," Zh. Vychisl. Mat. Mat. Fiz., 7, No. 4, 910-914 (1967).
3. V. B. Glasko, M. V. Zakharov, and A. Ya. Kolp, "Return of heat flux to body surface for nonlinear heat conduction based on regularization method," Inzh.-Fiz. Zh., 29, No. 1, 60-62 (1975).
4. O. M. Alifanov, "Regularizing schemes for solving inverse heat-conduction problems," Inzh.-Fiz. Zh., 24, No. 2, 324-333 (1973).

5. O. M. Alifanov, State and Likely Developments of Inverse Heat-Transfer Problems in Investigating Thermal Processes and Designing Technical Systems [in Russian], Preprint of Scientific Soviet on Heat- and Mass-Transfer Processes in Technological Processes, Minsk (1977).
6. A. N. Tikhonov, V. B. Glasko, N. I. Kulik, and I. N. Shklyarov, "Inverse problem of heat-conduction theory," Zh. Vychisl. Mat. Mat. Fiz., 19, No. 3, 768-774 (1979).
7. N. B. Vargaftik, Thermophysical Properties of Materials [in Russian], Tekhénergoizdat, Moscow–Leningrad (1956).
8. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1966).
9. A. A. Samarskii and E. S. Nikolaev, Methods of Solving Grid Equations [in Russian], Nauka, Moscow (1978).
10. B. M. Budak and F. P. Vasil'ev, Approximate Methods of Solving Optimal-Control Problems (Lecture Texts) [in Russian], Moscow State University, Moscow (1969).
11. É. Gudremon, Special Steels [in Russian], Vol. 1, Metallurgiya, Moscow (1966).
12. N. I. Kobasko, "Computer investigation of thermal processes in quenching steel," Metalloved. Term. Obrab. Met., No. 10, 8-13 (1976).
13. K. Z. Shepelyakovskii, I. N. Shklyarov, and V. D. Kal'ner, "Regions of application of simultaneous and continuously sequential heating in surface quenching," Elektrotermiya, No. 73-74, 51 (1968).
14. I. N. Shklyarov and F. I. Shor, "Resistance to small plastic deformations of beams subjected to inductional surface quenching," Vestn. Mashinostr., No. 12, 59-63 (1967).
15. Gulyaev, Metals Science [in Russian], Metallurgiya, Moscow (1977).
16. A. A. Popov and L. E. Popova, Isometric and Thermokinetic Decay Diagrams of Supercooled Austenite [in Russian], Metallurgiya, Moscow (1965).

KINETICS OF THE REMOVAL OF LIQUID FROM CAPILLARY-POROUS BODIES IN A FLUIDIZED BED UNDER NONISOTHERMAL CONDITIONS

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A theory of mass transfer in capillary-porous bodies is proposed which allows for thermogradient transfer of a bound substance in liquid form. The results obtained are used to calculate the process of drying of ceramic articles in a fluidized bed.

Existing methods of calculating processes of drying of capillary-porous bodies under isothermal and nonisothermal conditions are based mainly on analytical solutions of the system of differential equations of heat and mass transfer known from the phenomenological theory of irreversible processes [1, 2]. The main obstacle to the wide use of these equations is the considerable nonlinearity of the problem – the dependence of the kinetic coefficients appearing in them on the concentration of the bound substance and the temperature [3-5]. Under isothermal drying conditions (in the case of bodies of small size), calculations with allowance for the dependence $a_m = f(\bar{u}, t)$ are made by zonal methods [4]. In the presence of a temperature gradient within the material being dried, the number of criteria determining the kinetics of the process grows considerably [6], and it becomes impossible to use the zonal method of calculation. In such cases one artificially separates the heat- and mass-exchange processes and allows for the influence of the temperature field on the kinetics of the mass transfer using functions obtained from experiment for the relation between the volumetric-mean concentration and temperature [7], which are subsequently used in calculations of transfer processes in systems having a solid phase under quasi-isothermal conditions. However, numerous experimental data give evidence of temperature gradients which exert considerable influence on the kinetics of the drying process [8-10].

Moreover, it should be noted that a phenomenological examination of the stated problem does not allow one to characterize the fluxes of the bound substance in a porous body.

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